Computers and Mathematics with Applications 62 (2011) 2330–2335

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Perturbation of solutions of ordinary linear homogeneous differential equations of the second order

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ARTICLE INFO

Article history:
Received 2 March 2009
Received in revised form 9 July 2011
Accepted 10 July 2011

Keywords:
Differential equation
Perturbation
Intermediate equation
Oscillatory solutions

ABSTRACT

In this paper we give a method for peturbation of solutions of linear homogeneous differential equation of the second order.

1. Introduction

We apply perturbation by means of force $z(x)$ with various aims (oscillatoriness, monotonicity, limitedness by given boundaries, transformation of oscillatoriness into monotonicity and vice versa, slow rise, periodicity, transformation from oscillatoriness into periodicity, locations of zeros, approximate elimination of non-linearities, useful reduction or increasing of order, clearance of non-homogeneity, various Sturm theorems, oscillations, their zeros, amplitudes, frequencies etc). Specifically, it is done here in order to obtain oscillatory solutions; we apply amended [1–8], Sturm theorems [9], with exact locations of zeros.

Let there be a linear homogeneous differential equation of the second order

$$y'' + a(x)y' + b(x)y = 0 \tag{1}$$

where $a(x)$ and $b(x)$ are given continuous functions. Then, the equation has continuous solutions of a trigonometric (elliptic or hyperbolic) type [3]. Let y_1, y_2 be a fundamental set of solutions, considered known. A simple problem arises:

Is there a linear homogeneous differential equation of the second order with fundamental solutions:

$$\left.\begin{aligned} v_1 &= y_1 + z(x) \\ v_2 &= y_2 + z(x) \end{aligned}\right\} \tag{2}$$

and when does it exist, if a force $z(x)$ is applied to all the solutions $y_{1,2}(x)$.

Specifically, when will the equation under the force $z(x)$ have oscillatory solutions, and then what are the locations of zeros? Finally, what is the inverse equation for perturbation $z(x)$?

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0898-1221/$ – see front matter
doi:10.1016/j.camwa.2011.07.019